
CRYPTANALYSIS OF SELF -INVERTIBLE KEY GENERATION FOR AFFINE HILL CIPHER AND DIGRAPH AFFINE HILL CIPHER

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Abstract:

Bibhudendra Acharya in [15] proposed the concepts of Novel Methods of Generating Self-Invertible Matrix for Hill Cipher Algorithm. In this paper introduce Digraph Affine Hill cipher and discuss about cryptanalysis of Affine Hill cipher and Digraph Affine Hill cipher using Self-Invertible Matrix.

Keywords:

Self-Invertible Matrix,
Encryption algorithm,
decryption algorithm,
Affine Hill cipher,
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1. Introduction

Today, in the information age, the need to protect communications from prying eyes is greater than ever before [5]. Cryptography, the science of encryption and it plays a central role in many aspects of our daily lives like security of ATM cards, computer passwords, sending emails, e-commerce. Conventional Encryption is called symmetric encryption or single key encryption. It may be similarly divided into classes of classical techniques and modern strategies. The hallmark of traditional encryption is that the cipher or key to the set of rules is shared, i.e., regarded with the aid of the parties involved inside the secured communication exchange. Substitution cipher is one of the simple components of classical ciphers. A substitution cipher is a technique of encryption by way of which devices of plaintext are substituted with cipher text according to a regular system; the units may be single letters (the most common), pairs of letters, triplets of letters, combinations of the above, and so on. The receiver decipheres the text by using appearing an inverse substitution [3]. The units of the plaintext are retained within the identical sequence as inside the cipher text, but the devices themselves are altered. There are some of special sorts of substitution cipher. If the cipher operates on single letters, it's miles termed easy substitution cipher; a cipher that operates on large organizations of letters is named polygraphic. A monoalphabetic cipher uses constant substitution over the whole message, whereas a polyalphabetic cipher makes use of a number of substitutions at one-of-a-kind instances in the message— along with homophones, where a unit from the plaintext is mapped to considered one of several possibilities inside the cipher text. The inverse of the matrix used for encrypting the plaintext does not constantly exist. So, if the matrix is not invertible, the encrypted text can't be decrypted. In the self-invertible matrix generation method, the matrix used for the encryption is itself self-invertible. So, at the time of decryption, we want now not to find inverse of the matrix. Moreover, this technique gets rid of the computational complexity concerned in finding inverse of the matrix at the same time as decryption. Bibhudendra Acharya in [5], generated Self-Invertible Matrix and used in Hill cipher. In this paper we discuss about cryptanalysis of Affine Hill cipher and Digraph Affine Hill cipher using Self-Invertible Matrix algorithm.

2. Preliminaries:

2.1. Self-Invertible Matrix: A square matrix A is said to be Self-Invertible Matrix if $A^{-1} = A$.

2.2. The Affine Hill cipher:

The Affine Hill cipher extends the concept of Hill cipher by mixing it with a nonlinear affine transformation [2]. The Affine Hill cipher is an application linear algebra. It is one the block cipher to encrypt and decrypt the messages using matrix key and its inverse and it is a symmetric key algorithm. So the encryption expression will have the form of $Q = (KP + R)(\text{mod } m)$. All operations are performed over Z_m . Where R is column vector over Z_m . It should satisfy $\gcd(\det K(\text{mod } m), m) = 1$ and P is the plain text, Q is the cipher text.

Encryption:

$$Q = \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{pmatrix} P + R \pmod{m}$$

Decryption:

$$P = \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{pmatrix}^{-1} (Q - R) \pmod{m}$$

3. A General method of generating an even order self-invertible matrix under modulation of a prime number:

Let $K = \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{pmatrix}$ be a matrix of order n and p be a prime number.

Then $K = \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{pmatrix}$ be an $n \times n$ self-invertible matrix partitioned into

$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}$ where n is even and K_{11}, K_{12}, K_{21} and K_{22} are matrices of order $\frac{n}{2} \times \frac{n}{2}$ each.

Construction of an even self-invertible matrix

- Select any arbitrary $\frac{n}{2} \times \frac{n}{2}$ matrix K_{22}
- Obtain $K_{11} = -K_{22} \pmod{p}$
- If take $K_{12} = c(I - K_{11}) \pmod{p}$ then $K_{21} = \frac{(I + K_{11})}{c} \pmod{p}$ where c is a scalar constant and I is the identity matrix.
- Form the matrix completely

3.1.Example:

Let $p=37$, let $n=2$, let $K_{22} = 4$, let $c=2$

$$K_{11} = -K_{22} \pmod{p} \Rightarrow K_{11} = -4 \pmod{37} \Rightarrow K_{11} = 33$$

$$K_{12} = 2(I - K_{11}) \pmod{p} \Rightarrow K_{12} = 2(1 - 33) \pmod{37} \Rightarrow K_{12} = 10$$

$$K_{21} = \frac{(I + K_{11})}{c} \pmod{p} \Rightarrow K_{21} = \frac{(1 + 33)}{2} \pmod{37} \Rightarrow K_{21} = 17 \pmod{37}$$

Therefore $K = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \pmod{37}$ is an even order self-invertible matrix.

3.2.Example:

Let $p=37$, let $n=2$, let $K_{22} = 4$, let $c=1$

$$K_{11} = -K_{22} \pmod{p^2} \Rightarrow K_{11} = -4 \pmod{37^2} \Rightarrow K_{11} = 1365$$

$$K_{12} = (I - K_{11}) \pmod{p^2} \Rightarrow K_{12} = (1 - 1365) \pmod{37^2} \Rightarrow K_{12} = 5$$

$$K_{21} = \frac{(I + K_{11})}{c} \pmod{p^2} \Rightarrow K_{21} = (1 + 1365) \pmod{37^2} \Rightarrow K_{21} = 1366 \pmod{37^2}$$

Therefore $K = \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix} \pmod{37^2}$ is an even order self-invertible matrix.

3.3.Example:

Let $p=37$, let $n=4$, let $K_{22} = \begin{pmatrix} 10 & 2 \\ 3 & 4 \end{pmatrix}$, let $c=1$

$$K_{11} = -K_{22} \pmod{p} \Rightarrow K_{11} = -\begin{pmatrix} 10 & 2 \\ 3 & 4 \end{pmatrix} \pmod{37} \Rightarrow K_{11} = \begin{pmatrix} 27 & 35 \\ 34 & 33 \end{pmatrix} \pmod{37}$$

$$K_{12} = c(I - K_{11}) \pmod{p} \Rightarrow K_{12} = (I - K_{11}) \pmod{37} \Rightarrow K_{12} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} -$$

$$\begin{pmatrix} 27 & 35 \\ 34 & 33 \end{pmatrix} \pmod{37} = \begin{pmatrix} 11 & 2 \\ 3 & 5 \end{pmatrix}$$

$$K_{21} = \frac{(I + K_{11})}{c} \pmod{p} \Rightarrow K_{21} = \begin{pmatrix} 28 & 35 \\ 34 & 34 \end{pmatrix}$$

Therefore $K = \begin{pmatrix} 27 & 35 & 11 & 2 \\ 34 & 33 & 3 & 5 \\ 28 & 35 & 10 & 2 \\ 34 & 34 & 3 & 4 \end{pmatrix} \pmod{37}$ is an even order self-invertible matrix.

4. An even order self-invertible matrix on Affine Hill Cipher:

Prime modulus generates large key space than a composite modulus and taking q is prime number. The plaintext column vector \mathbf{P} is encrypted as $\mathbf{Q} = (\mathbf{K}\mathbf{P} + \mathbf{R}) \pmod{q}$ in which \mathbf{Q} is the cipher text column vector, \mathbf{K} is an even order $m \times m$ key matrix in Z_q and The cipher text \mathbf{Q} is decrypted as

$\mathbf{P} = \mathbf{K}(\mathbf{Q} - \mathbf{R}) \pmod{q}$ where $\mathbf{R} = \mathbf{K}_i (1 \leq i \leq m)$ is one of the column vector \mathbf{K} over Z_q and It should satisfy $\text{g.c.d}(\det \mathbf{K} \pmod{q}, q) = 1$. If the length of the plain text is odd then add space letter to given plain text so that the length of the plain text is even.

4.1. Encryption algorithm

- Step1: Generate even order self-invertible matrix key \mathbf{K} .
- Step2: Calculate $\mathbf{Q} = (\mathbf{K}\mathbf{P} + \mathbf{R}) \pmod{q}$
- Step3: Write Cipher text

4.2. Decryption algorithm:

- Step1: Input Keys \mathbf{K}, \mathbf{R}
- Step2: Calculate $\mathbf{P} = \mathbf{K}(\mathbf{Q} - \mathbf{R}) \pmod{q}$
- Step3: Write Plain text

4.3. An example of even order self-invertible matrix on Affine Hill Cipher:

In the 37-letter alphabet in which A-Z have numerical equivalent 0-25, 0-9 have numerical equivalent 26-35, space=36. And $q=37$.

Now consider plain text "YK17"

$$\mathbf{P} = \begin{pmatrix} Y \\ K \\ 1 \\ 17 \end{pmatrix} \text{ is block of plain text. } \mathbf{P} = \begin{pmatrix} 24 \\ 10 \\ 27 \\ 33 \end{pmatrix} \text{ is block of plain text.}$$

$$\text{Taking keys from example 3.2, we get } \mathbf{K} = \begin{pmatrix} 27 & 35 & 11 & 2 \\ 34 & 33 & 3 & 5 \\ 28 & 35 & 10 & 2 \\ 34 & 34 & 3 & 4 \end{pmatrix} \text{ and } \mathbf{R} = \begin{pmatrix} 27 \\ 34 \\ 28 \\ 34 \end{pmatrix}$$

Encryption:

$$\mathbf{Q} = (\mathbf{K}\mathbf{P} + \mathbf{R}) \pmod{37}$$

$$\mathbf{Q} = \left(\begin{pmatrix} 27 & 35 & 11 & 2 \\ 34 & 33 & 3 & 5 \\ 28 & 35 & 10 & 2 \\ 34 & 34 & 3 & 4 \end{pmatrix} \begin{pmatrix} 24 \\ 10 \\ 27 \\ 33 \end{pmatrix} + \begin{pmatrix} 27 \\ 34 \\ 28 \\ 34 \end{pmatrix} \right) \pmod{37}$$

$$\mathbf{Q} = \begin{pmatrix} 19 \\ 20 \\ 17 \\ 34 \end{pmatrix} \pmod{37}$$

Cipher text="TUR8"

Decryption: $P=K(Q-R) \pmod{37}$

$$R = \begin{pmatrix} 27 & 35 & 11 & 2 \\ 34 & 33 & 3 & 5 \\ 28 & 35 & 10 & 2 \\ 34 & 34 & 3 & 4 \end{pmatrix} \begin{pmatrix} 19 \\ 20 \\ 17 \\ 34 \end{pmatrix} \pmod{37} = \begin{pmatrix} 24 \\ 10 \\ 27 \\ 33 \end{pmatrix}$$

Which gives original plain text is "YK17"

Now consider another plain text "YEAR2017"

$P = \begin{pmatrix} Y & A & 2 & 1 \\ E & R & 0 & 7 \end{pmatrix}$ is block of plain text. $P = \begin{pmatrix} 24 & 0 & 28 & 27 \\ 4 & 17 & 26 & 33 \end{pmatrix}$ is block of plain text.

Taking keys from example 3.1, we get $K = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix}$, $R = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

Encryption: $Q = (KP+R) \pmod{37}$

$$Q_1 = \left(\begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 24 \\ 4 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \end{pmatrix} \right) \pmod{37} = \begin{pmatrix} 28 \\ 21 \end{pmatrix}$$

$$Q_2 = \left(\begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 17 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \end{pmatrix} \right) \pmod{37} = \begin{pmatrix} 32 \\ 35 \end{pmatrix}$$

$$Q_3 = \left(\begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 28 \\ 26 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \end{pmatrix} \right) \pmod{37} = \begin{pmatrix} 10 \\ 29 \end{pmatrix}$$

$$Q_4 = \left(\begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 27 \\ 33 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \end{pmatrix} \right) \pmod{37} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$$

Cipher text $Q = (Q_1, Q_2, Q_3, Q_4) = "2V69K3KD"$

Decryption: $P=K(Q-R) \pmod{37}$

$$P_1 = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 28 \\ 21 \end{pmatrix} \pmod{37} = \begin{pmatrix} 24 \\ 4 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 32 \\ 35 \end{pmatrix} \pmod{37} = \begin{pmatrix} 0 \\ 17 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 10 \\ 29 \end{pmatrix} \pmod{37} = \begin{pmatrix} 28 \\ 26 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix} \begin{pmatrix} 10 \\ 3 \end{pmatrix} \pmod{37} = \begin{pmatrix} 27 \\ 33 \end{pmatrix}$$

This gives original plain text $P = (P_1, P_2, P_3, P_4) = "YEAR2017"$

5. Cryptanalysis of known plaintext attack when m is known on Affine Hill cipher:

The Affine Hill Cipher, the opponent has to know the values of the size m of the plaintext block size n of the cipher text block and the constant vector b to determine the matrix of the transformation even for a known plaintext attack. Let us assume that the opponent has determined the value of the m being used. Let m be distinct plaintext-cipher text pairs, say, $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and $y_j = (y_{1j}, y_{2j}, \dots, y_{mj})$, $1 \leq j \leq m$, such that $y_j = ek(x_j) + b$. Define $m \times m$ matrices $P = (x_{ij})$ and $Q = (y_{ij})$. Whenever K is invertible in the encryption equation $Q = KP + R$, one has to choose a different continuous pair of plaintext-cipher text pairs.

5.1. Algorithm for known plain-text attack when m is known

Step1: Let $P = (P_1, P_2, \dots, P_n)$ be a block of plain text. Where $P_i = \begin{pmatrix} p_{1i} \\ p_{2i} \\ \vdots \\ p_{mi} \end{pmatrix}$.

Let $Q = (C_1, C_2, \dots, C_n)$ be a block of cipher text. Where $C_i = \begin{pmatrix} c_{1i} \\ c_{2i} \\ \vdots \\ c_{mi} \end{pmatrix}$

Select continuous pairs of plaintext and their corresponding cipher text are (P_i, C_i) for $i=1, 2, 3, \dots, n$.

Step 2: The known plaintext – cipher text pairs gives us a system of equations of the form

$[C_i - C_j] = K[P_i - P_j]$ for $1 \leq i \neq j \leq m$, we get general equation $C = KP$

Step 3: Determine $K = P^{-1}C \pmod{q}$

Step 4: Determine R, substitute K, P, C in $C = (KP + R) \pmod{q}$

5.2. Example for cryptanalysis when m=2 is known

Taking from Example 4.3, we get plaintext blocks are $P_1 = \begin{pmatrix} 24 \\ 4 \end{pmatrix}$, $P_2 = \begin{pmatrix} 0 \\ 17 \end{pmatrix}$, $P_3 = \begin{pmatrix} 28 \\ 26 \end{pmatrix}$, corresponding cipher text

blocks are $C_1 = \begin{pmatrix} 28 \\ 21 \end{pmatrix}$, $C_2 = \begin{pmatrix} 32 \\ 35 \end{pmatrix}$, $C_3 = \begin{pmatrix} 10 \\ 29 \end{pmatrix}$

then $(C_1 - C_2 \quad C_2 - C_3) = K(P_1 - P_2 \quad P_2 - P_3)$

$$\begin{pmatrix} 33 & 22 \\ 23 & 6 \end{pmatrix} = K \begin{pmatrix} 24 & 9 \\ 24 & 28 \end{pmatrix} \Rightarrow K = \begin{pmatrix} 33 & 22 \\ 23 & 6 \end{pmatrix} \begin{pmatrix} 24 & 9 \\ 24 & 28 \end{pmatrix}^{-1} = \begin{pmatrix} 33 & 10 \\ 17 & 4 \end{pmatrix}$$

$$Q = (KP + R) \Rightarrow R = Q - KP = \begin{pmatrix} 28 \\ 21 \end{pmatrix} - \begin{pmatrix} 18 \\ 17 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

To overcome this we make use of Digraph Affine Hill Cipher

6. Digraph Affine Hill Cipher:

The Digraph Affine Hill cipher is an application linear algebra. It is one the block cipher to encrypt and decrypt the messages using matrix key and its inverse and it is a symmetric key algorithm. The plaintext column vector P is encrypted as $Q = (PK + R) \pmod{q^2}$ in which Q is the cipher text column vector, K is an $m \times m$ key matrix in Z_{q^2} and The cipher text Q is decrypted as $P = (Q - R)K^{-1} \pmod{q^2}$ where $R = K_i (1 \leq i \leq m)$ is one of the column vector K over Z_{q^2} and It should satisfy $\text{g.c.d}(\det K \pmod{q}, q) = 1$. Where q is prime number. Let n be an even length plaintext. If n is not even length of plain text add space letter then $n+1$ will be even. Let plain text $P = (P_1, P_2, P_3, \dots, P_{n/2})$ and cipher text $Q = (Q_1, Q_2, Q_3, \dots, Q_{n/2})$. In Digraph Affine cipher, plaintext and cipher text messages are 2-letter blocks, called digraphs. i.e plaintext is combined into 2-letter segments. If the entire plaintext has odd number of letters, then in order to obtain a whole number of digraphs add an extra letter (space) at the end.

$$P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$$

(1)

where $i=0, 2, 4, 6, \dots, n-2$, x_i is the numerical equivalent of the i^{th} letter of plaintext in the digraph Cipher text is split into 2-letter segments.

$$Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$$

(2)

where $i=0, 2, 4, 6, \dots, n-2$, y_i is the numerical equivalent of the i^{th} letter of cipher text in the digraph. In decryption process cipher text is combined into 2-letter segments using (2), after decryption plaintext is split into 2-letter segments using (1)

Encryption:

combined into 2 – letter segments $P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$ where $i=0, 2, 4, 6, \dots, n-2$

$$Q_j = (P_j \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{pmatrix} + R) \pmod{q^2} \text{ where } j=1,2,\dots,\frac{n}{2}$$

split into 2-letter segments $Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

Decryption:

combine into 2 – letter segments $Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

$$P_j = (Q_j - R) \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{pmatrix}^{-1} \pmod{q^2} \text{ where } j=1,2,\dots,\frac{n}{2}$$

split into 2-letter segments $P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

7.An even order self-invertible matrix on Digraph Affine Hill Cipher:

The plaintext column vector \mathbf{P} is encrypted as $\mathbf{Q} = (\mathbf{K}\mathbf{P} + \mathbf{R}) \pmod{q^2}$ in which \mathbf{Q} is the cipher text column vector, \mathbf{K} is an even order $m \times m$ key matrix in $\mathbb{Z}q^2$ and The cipher text \mathbf{Q} is decrypted as

$\mathbf{P} = \mathbf{K}(\mathbf{Q} - \mathbf{R}) \pmod{q^2}$ where $\mathbf{R} = \mathbf{K}_i (1 \leq i \leq m)$ is one of the row vector \mathbf{K} over $\mathbb{Z}q^2$ and It should

satisfy $\text{g.c.d}(\det \mathbf{K} \pmod{q}, q) = 1$. Let plain text $\mathbf{P} = (P_1, P_2, P_3, \dots, P_{n/2})$ and cipher text

$\mathbf{Q} = (Q_1, Q_2, Q_3, \dots, Q_{n/2})$.

Encryption:

The plaintext is combining into 2-letter segments

ie combine into 2 – letter segments $P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

$$Q = (PK + R) \pmod{q^2}$$

split into 2-letter segments $Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

It can be written as $Q_{\frac{i}{2}+1} = (y_{i+1}, y_{i+2})$ where $i=0,2,4,6,\dots,n-2$

Decryption:

combine into 2 – letter segments $Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

$$P = (Q - R)K \pmod{q^2}$$

split into 2-letter segments $P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$ where $i=0,2,4,6,\dots,n-2$

It can be written as $P_{\frac{i}{2}+1} = (x_{i+1}, x_{i+2})$ where $i=0,2,4,6,\dots,n-2$.

7.1.An example of even order self-invertible matrix on Digraph Affine Hill Cipher:

In the 37-letter alphabet in which A-Z have numerical equivalent 0-25, 0-9 have numerical equivalent 26-35, space=36. And $q=37$.

Plain text=YEAR2017, Length of the plain text $n=8$, $\mathbf{P} = (P_1, P_2, P_3, P_4)$ where $P_1=(24, 4)$, $P_2=(0, 17)$, $P_3=(28, 26)$, $P_4=(27, 33)$

Taking keys from example 3.2, we get $\mathbf{K} = \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix}$, $\mathbf{R} = (1365 \ 5)$

Encryption:

combine into 2 – letter segments $P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$ where $i=0,2,4,6$

$$\text{For } i=0, P_1 = (37x_1 + x_2) \pmod{37^2} \Rightarrow P_1 = ((24)(37) + 4) \pmod{37^2} = 892 \pmod{37^2}$$

$$\text{For } i=2, P_2 = (37x_3 + x_4) \pmod{37^2} \Rightarrow P_2 = ((0)(37) + 17) \pmod{37^2} = 17 \pmod{37^2}$$

$$\text{For } i=4, P_3 = (37x_5 + x_6) \pmod{37^2} \Rightarrow P_3 = ((28)(37) + 26) \pmod{37^2} = 1062 \pmod{37^2}$$

$$\text{For } i=6, P_4 = (37x_7 + x_8) \pmod{37^2} \Rightarrow P_4 = ((27)(37) + 33) \pmod{37^2} = 1032 \pmod{37^2}$$

After combine into 2 – letter segments, $P_1 = 892, P_2 = 17, P_3 = 1062, P_4 = 1032$

$$Q = (PK + R) \pmod{37^2}$$

$$Q = \left((892 \ 17) \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix} + (1365 \ 5) \right) \pmod{37^2} = (484 \ 426)$$

$$Q = \left((1062 \ 1032) \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix} + (1365 \ 5) \right) \pmod{37^2} = (1354 \ 1229)$$

Split into 2-letter segments $Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$ where $i=0,2,4,6$

$$\text{For } i=0, Q_1 = 484 = (y_1 37 + y_2) \pmod{37^2} \Rightarrow Q_1 = (13(37) + 3) \pmod{37^2} \Rightarrow Q_1 = (13, 3)$$

$$\text{For } i=2, Q_2 = 426 = (y_3 37 + y_4) \pmod{37^2} \Rightarrow Q_2 = (11(37) + 19) \pmod{37^2} \Rightarrow Q_2 = (11, 19)$$

$$\text{For } i=4, Q_3 = 1354 = (y_5 37 + y_6) \pmod{37^2} \Rightarrow Q_3 = (36(37) + 22) \pmod{37^2} \Rightarrow Q_3 = (36, 22)$$

$$\text{For } i=6, Q_4 = 1229 = (y_7 37 + y_8) \pmod{37^2} \Rightarrow Q_4 = (33(37) + 8) \pmod{37^2} \Rightarrow Q_4 = (33, 8).$$

∴ Cipher text $Q = (Q_1, Q_2, Q_3, Q_4) = \text{"NDLT W7I"}$

Decryption:

combine into 2 – letter segments $Q_{\frac{i}{2}+1} = (y_{i+1}q + y_{i+2}) \pmod{q^2}$ where $i=0,2,4,6$

$$\text{For } i=0, Q_1 = (y_1 37 + y_2) \pmod{37^2} \Rightarrow Q_1 = (13(37) + 3) \pmod{37^2} = 484 \pmod{37^2}$$

$$\text{For } i=2, Q_2 = (y_3 37 + y_4) \pmod{37^2} \Rightarrow Q_2 = (11(37) + 19) \pmod{37^2} = 426 \pmod{37^2}$$

$$\text{For } i=4, Q_3 = (y_5 37 + y_6) \pmod{37^2} \Rightarrow Q_3 = (36(37) + 22) \pmod{37^2} = 1354 \pmod{37^2}$$

$$\text{For } i=6, Q_4 = (y_7 37 + y_8) \pmod{37^2} \Rightarrow Q_4 = (33(37) + 8) \pmod{37^2} = 1229 \pmod{37^2}$$

After combine into 2 – letter segments, $Q_1 = 484, Q_2 = 426, Q_3 = 1354, Q_4 = 1229$

$$P = (Q - R)K \pmod{37^2}$$

$$P = \left((488 \ 430) \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix} \right) \pmod{37^2} = (893 \ 17)$$

$$P = \left((8701224) \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix} \right) \pmod{37^2} = (10621032)$$

Split into 2 – letter segments $P_{\frac{i}{2}+1} = (x_{i+1}q + x_{i+2}) \pmod{q^2}$ where $i=0,2,4,6$

$$\text{For } i=0, P_1 = 892 \Rightarrow P_1 = ((24)(37) + 4) \pmod{37^2} \Rightarrow P_1 = (24, 4)$$

$$\text{For } i=2, P_2 = 17 \Rightarrow P_2 = ((0)(37) + 17) \pmod{37^2} \Rightarrow P_2 = (0, 17)$$

$$\text{For } i=4, P_3 = 1062 \Rightarrow P_3 = ((28)(37) + 26) \pmod{37^2} \Rightarrow P_3 = (28, 26)$$

$$\text{For } i=6, P_4 = 1032 \Rightarrow P_4 = ((27)(37) + 33) \pmod{37^2} \Rightarrow P_4 = (27, 33)$$

It gives original plaintext = "YEAR2017"

8. Cryptanalysis of known plaintext attack when m is known on Digraph Affine Hill cipher:

The Digraph Affine Hill Cipher, the opponent has to know the values of the size m of the plaintext block size n of the cipher text block and the constant vector b to determine the matrix of the transformation even for a known plaintext attack. Let us assume that the opponent has determined the value of the m being used. Let m be distinct plaintext-cipher text pairs, say, $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and $y_j = (y_{1j}, y_{2j}, \dots, y_{mj})$, $1 \leq j \leq m$, such that $y_j = ek(x_j) + b$. Define $m \times m$ matrices $P = (x_{ij})$ and $Q = (y_{ij})$. Whenever K is invertible in the

encryption equation $Q=PK+R$, one has to choose a different continuous pair of plaintext-cipher text pairs.

8.1. Example for cryptanalysis when $m=2$ is known

Taking from Example 7.1, we get plaintext blocks are $P_1=(24,4)$, $P_2=(0,17)$, $P_3=(28,26)$, $P_4=(27,33)$, corresponding cipher text blocks are $Q_1 = (13, 3)$, $Q_2 = (11, 19)$, $Q_3 = (36, 22)$, $Q_4 = (33, 8)$.

By using 5.1. Algorithm for known plain-text attack when m is known then

$$\begin{pmatrix} Q_1 - Q_2 \\ Q_2 - Q_3 \end{pmatrix} = \begin{pmatrix} P_1 - P_2 \\ P_2 - P_3 \end{pmatrix} K$$

$$\begin{pmatrix} 2 & 21 \\ 12 & 34 \end{pmatrix} = K \begin{pmatrix} 24 & 24 \\ 9 & 28 \end{pmatrix} \Rightarrow K = \begin{pmatrix} 24 & 24 \\ 9 & 28 \end{pmatrix}^{-1} \begin{pmatrix} 33 & 22 \\ 23 & 6 \end{pmatrix} \pmod{37} = \begin{pmatrix} 30 & 18 \\ 4 & 6 \end{pmatrix}$$

But In example 7.1 we used the key $K = \begin{pmatrix} 1365 & 5 \\ 1366 & 4 \end{pmatrix}$

So, By using this method we avoid known plain text attack when m is known.

9. CONCLUSION

In this proposed system introduce Digraph Affine Hill cipher which is more secure than Affine Hill Cipher. These strategies envelop less computational complexity as inverse of the matrix is not required while decrypting in Affine Hill Cipher and Digraph Affine Hill cipher. Digraph Affine Hill cipher avoids known plain text attack.

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